Single Pure - Completing The Square

Patrons are reminded that if they see an expression such as $x^2 - 5 - 3x + ax + kx^2 + \pi$, the x^2, x and 'constant' terms can be collected by factoring to yield $(1 + k)x^2 + (a - 3)x + (\pi - 5)$. Thank you for listening.

1. Complete the square in the following quadratic expressions.

(a) $x^2 + 8x + 10$. $(x + 4)^2 - 6$

(b) $x^2 - 20x + 110$. $(x - 10)^2 + 10$

(c) $x^2 - 6x + 9$. $(x - 3)^2$

(d) $x^2 + 3x - 1$. $(x + \frac{3}{2})^2 - \frac{13}{4}$

(e) $x^2 - x + \frac{2}{3}$. $(x - \frac{1}{2})^2 + \frac{5}{12}$

(f) $3x^2 + 18x - 1$. $3(x + 3)^2 - 28$

(g) $5x^2 - 20x - 3$. $5(x - 2)^2 - 23$

(h) $-y^2 + 8y - 20$. $-(y - 4)^2 - 4$

(i) $2x^2 + x - 3$. $2(x + \frac{1}{4})^2 - \frac{25}{8}$

(j) $3x^2 + 4x$. $3(x + \frac{2}{3})^2 - \frac{4}{3}$

(k) $-x^2 + 5x + \frac{1}{3}$. $-(x - \frac{5}{2})^2 + \frac{79}{12}$

(l) $-2z^2 + \frac{1}{2}z - \frac{5}{4}$. $-2(z - \frac{1}{8})^2 - \frac{39}{32}$

(m) $x^2 + ax + x + 3$. $\left(x + \frac{a+1}{2}\right)^2 + \frac{11-a^2-2a}{4}$

(n) $ax^2 + bx + c$. $a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$

(o) $-2ax^2 - ax + \frac{1}{a}$. $-2a\left(x + \frac{1}{4}\right)^2 + \frac{a^2+8}{8a}$

(p) $3x^2 + \pi x - 2x$. $3\left(x + \frac{\pi-2}{6}\right)^2 - \frac{(\pi-2)^2}{12}$

(q) $kz^2 + z^2 + kz - 1$. $(k + 1)\left(z + \frac{k}{2(k+1)}\right)^2 - \frac{(k+2)^2}{4(k+1)}$

2. By completing the square find the vertex of the following curves and state whether it is a maximum or minimum.

(a) $y = x^2 - 14x + 11$. $(7, -38)$, min

(b) $y = x^2 + 5x$. $\left(-\frac{5}{2}, -\frac{25}{4}\right)$, min

(c) $y = x^2 + 4x + 4$. $(-2, 0)$, min

(d) $y = x^2 - 3x + 8$. $\left(\frac{3}{2}, \frac{23}{4}\right)$, min

(e) $y = 2x^2 + 12x - 5$. $(-3, -23)$, min

(f) $y = -x^2 + 2x + a$. $(1, 1 + a)$, max

(g) $y = 3x^2 - x + 1$. $\left(\frac{1}{6}, \frac{11}{12}\right)$, min

(h) $y = x^2 + kx$. $\left(-\frac{k}{2}, -\frac{k^2}{4}\right)$, min

(i) $y = -2x^2 + ax - 3$. $\left(\frac{a}{4}, \frac{a^2-24}{8}\right)$, max

(j) $y = 3x^2 + rx + mx - 1$. $\left(-\frac{r+m}{6}, -\frac{(r+m)^2+12}{12}\right)$, min

3. By completing the square find the line of symmetry of the following curves.

(a) $y = x^2 + 12x - 3$. $x = -6$

(b) $y = x^2 - 7x$. $x = \frac{7}{2}$

(c) $y = -x^2 + 2x - 1$. $x = 1$

(d) $y = 2x^2 - x + 5$. $x = \frac{1}{4}$

(e) $y = 3x^2 + 5x - 2$. $x = -\frac{5}{6}$

(f) $y = -2x^2 + \frac{1}{a}x - 1$. $x = \frac{1}{4a}$

4. Solve the following equations by completing the square.

(a) $x^2 + 4x - 3 = 0$. $x = -2 \pm \sqrt{7}$

(b) $x^2 = 2x + 5$. $x = 1 \pm \sqrt{6}$

(c) $x^2 + x = 7$. $x = \frac{-1 \pm \sqrt{29}}{2}$

(d) $2x^2 - 8x - 1 = 0$. $x = \frac{4 \pm 3\sqrt{2}}{2}$

5. Sketch the following curves by completing the square.

(a) $y = x^2 + 3x - 5$.

(b) $y = 2x^2 + x - 7$.

6. Extending the concept. Find the minimum or maximum point(s) on the following graphs.

(a) $y = (x^2 - 1)^2 + 5$. $(\pm 1, 5)$, min

(b) $y = (4x^2 - 9)^2 - 1$. $(\pm \frac{3}{2}, -1)$, min

(c) $y = (x^2 + x - 6)^2 + k$. $(2, k)$ or $(-3, k)$, min

(d) $y = -(3x^2 - 10x - 8)^2 + 5$. $(4, 5)$ or $(-\frac{2}{3}, 5)$, max

(e) $y = (x^2 - px - qx + pq)^2 - pq$. $(p, -pq)$ or $(q, -pq)$, min

(f) $y = x^4 - 4x^2 + 1$. $(\pm\sqrt{2}, -2)$, min

(g) $y = -x^4 + 8x^2 - 9$. $(\pm 2, 7)$, max

(h) $y = (x^2 - 2x + 2)^2 + 10$. $(1, 11)$, min

7. Find the maximum value of $\dfrac{4}{x^2 - 6x + 1}$.